Model Question B.Sc. Sem-IV ①
Mathematics (GE-4)

Short answer type question.

1) Define a) Riemann Integral
b) Refinement of Partition
c) Oscillatory Sum.

2) Prove that every bounded constant function on $[a, b]$ is R-integrable.

3) Define i) a partially ordered set
~~ii) Define~~ a
ii) Maximal and minimal elements of a partially ordered set. Give examples.

4) a) Define a Lattice and Complete Lattice.
b) State Zorn's Lemma

5) Define a) Analytic Function
b) Harmonic Function
c) Uniform Continuity.

6). Show that the function $e^x(\cos y + i \sin y)$ is analytic and find its derivative.

7) If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, find the value of $A^2 - 4A + 3I$, where I is the unit matrix with proper dimension.

8) Find the adjoint of the matrix

$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{bmatrix}$$

9) Solve i) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

ii) $\frac{d^2y}{dx^2} + 4y = 0$

iii) $\frac{d^3y}{dx^3} - 3\frac{dy}{dx} + 2y = 0$.

10). Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2$.

Long answer type question.

1) Let f be a bounded function defined on $[a,b]$ and let m and M be the infimum and Supremum of f on $[a,b]$. Then prove that for every partition P of $[a,b]$

$$m(b-a) \leq L(P) \leq U(P) \leq M(b-a)$$

2) If f is monotonic on $[a,b]$, then prove that $f \in R[a,b]$.

3) Give an example of a set which is partially ordered but which is not totally ordered.

4) a) Consider the set $S = \{1,2,3,4,12\}$ and let the relation '$\leq$' be defined by $a \leq b$ if a divides b.
e.g. $3 \leq 12$ since 3 divides 12 etc.
Find the maximal element of S.

b) Give an example of partially ordered set which is not a Lattice.

5) ~~If~~ Prove that, if a function
$$f(z) = u(x,y) + iv(x,y)$$
is differentiable at any point $z = x+iy$, the partial derivatives u_x, v_x, u_y, v_y should exist and satisfy the equations
$$u_x = v_y, \quad u_y = -v_x.$$

6) Find the polar form of Cauchy-Riemann Equation.

78) Find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 6 & 7 & 9 \end{bmatrix}$$

8) i) If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 2 & 4 \end{bmatrix}$, P.T. $(AB)' = B'A'$

ii) verify that $\frac{1}{3}\begin{bmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ is orthogonal,

9) Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x$.

10) Solve,

$$\frac{dx}{x(y^2-z^2)} = \frac{dy}{y(z^2-x^2)} = \frac{dz}{z(x^2-y^2)}.$$